Math Virtual Learning

Calculus AB

Infinite Limits

April 30, 2020

Calculus AB
Lesson: April 30, 2020

Objective/Learning Target:
Lesson 4 Limits Review
Students will evaluate infinite limits.

Warm-Up:

Note: This is a review of 1st Semester Material. For more examples refer back to your 1st Semester notes.

Watch Video: Infinite Limits

Read Article: Infinite Limits

Notes:

THEOREM 1.14 Vertical Asymptotes

Let f and g be continuous on an open interval containing c. If $f(c) \neq 0$, $g(c) = 0$, and there exists an open interval containing c such that $g(x) \neq 0$ for all $x \neq c$ in the interval, then the graph of the function given by

$$h(x) = \frac{f(x)}{g(x)}$$

has a vertical asymptote at $x = c$.

Notes:

THEOREM 1.15 Properties of Infinite Limits

Let c and L be real numbers and let f and g be functions such that

$$\lim_{x \to c} f(x) = \infty \qquad \text{and} \qquad \lim_{x \to c} g(x) = L.$$

1. Sum or difference: $\lim_{x \to c} [f(x) \pm g(x)] = \infty$
2. Product: $\lim_{x \to c} [f(x)g(x)] = \infty, \quad L > 0$

   $\lim_{x \to c} [f(x)g(x)] = -\infty, \quad L < 0$
3. Quotient: $\lim_{x \to c} \dfrac{g(x)}{f(x)} = 0$

Similar properties hold for one-sided limits and for functions for which the limit of $f(x)$ as x approaches c is $-\infty$.

Examples:

Determine all vertical asymptotes of the graph of

$$f(x) = \frac{x^2 + 2x - 8}{x^2 - 4}.$$

Solution Begin by simplifying the expression, as shown.

$$f(x) = \frac{x^2 + 2x - 8}{x^2 - 4}$$

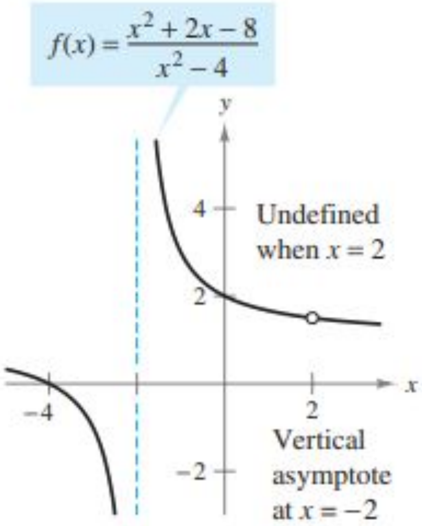
$$= \frac{(x + 4)\cancel{(x - 2)}}{(x + 2)\cancel{(x - 2)}}$$

$$= \frac{x + 4}{x + 2}, \quad x \neq 2$$

At all x-values other than $x = 2$, the graph of f coincides with the graph of $g(x) = (x + 4)/(x + 2)$. So, you can apply Theorem 1.14 to g to conclude that there is a vertical asymptote at $x = -2$, as shown in Figure 1.43. From the graph, you can see that

$$\lim_{x \to -2^-} \frac{x^2 + 2x - 8}{x^2 - 4} = -\infty \quad \text{and} \quad \lim_{x \to -2^+} \frac{x^2 + 2x - 8}{x^2 - 4} = \infty.$$

Note that $x = 2$ is *not* a vertical asymptote.



$f(x)$ increases and decreases without bound as x approaches -2.

Figure 1.43

Examples:

Find each limit.

$$\lim_{x\to 1^-} \frac{x^2 - 3x}{x - 1} \quad \text{and} \quad \lim_{x\to 1^+} \frac{x^2 - 3x}{x - 1}$$

Solution Because the denominator is 0 when $x = 1$ (and the numerator is not zero), you know that the graph of

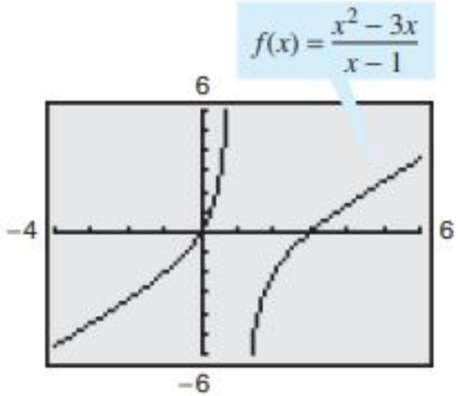
$$f(x) = \frac{x^2 - 3x}{x - 1}$$

has a vertical asymptote at $x = 1$. This means that each of the given limits is either ∞ or $-\infty$. A graphing utility can help determine the result. From the graph of f shown in Figure 1.44, you can see that the graph approaches ∞ from the left of $x = 1$ and approaches $-\infty$ from the right of $x = 1$. So, you can conclude that

$$\lim_{x\to 1^-} \frac{x^2 - 3x}{x - 1} = \infty$$ The limit from the left is infinity.

and

$$\lim_{x\to 1^+} \frac{x^2 - 3x}{x - 1} = -\infty.$$ The limit from the right is negative infinity.



f has a vertical asymptote at $x = 1$.
Figure 1.44

Practice:

1) Evaluate each of the following limits.

$$\lim_{x \to 0^+} \frac{6}{x^2} \qquad \lim_{x \to 0^-} \frac{6}{x^2} \qquad \lim_{x \to 0} \frac{6}{x^2}$$

2) Evaluate each of the following limits.

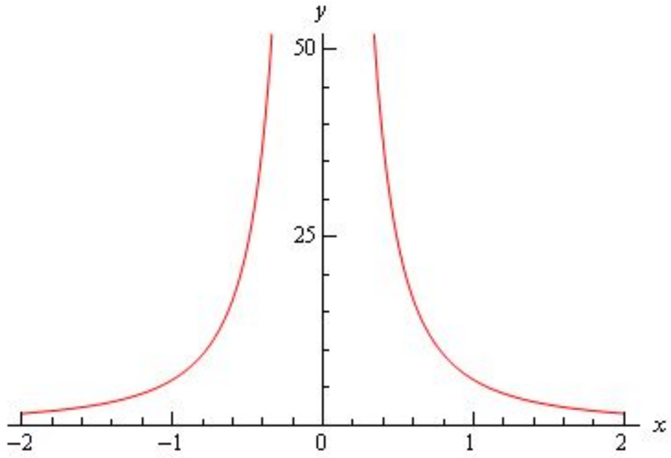
$$\lim_{x \to 4^+} \frac{3}{(4-x)^3} \qquad \lim_{x \to 4^-} \frac{3}{(4-x)^3} \qquad \lim_{x \to 4} \frac{3}{(4-x)^3}$$

Answer Key:

Once you have completed the problems, check your answers here.

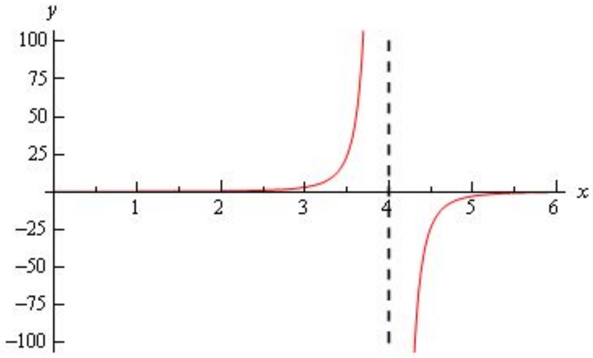
1)

$$\lim_{x \to 0^+} \frac{6}{x^2} = \infty \qquad \lim_{x \to 0^-} \frac{6}{x^2} = \infty \qquad \lim_{x \to 0} \frac{6}{x^2} = \infty$$



2)

$$\lim_{x \to 4^+} \frac{3}{(4-x)^3} = -\infty \qquad \lim_{x \to 4^-} \frac{3}{(4-x)^3} = \infty \qquad \lim_{x \to 4} \frac{3}{(4-x)^3} \text{ doesn't exist}$$



Additional Practice:

In your Calculus book read section 1.5 and complete problems 1, 11, 13, 19, 29, 33, 37, 39, and 43 on page 88

Interactive Practice:graphically

Interactive Practice: algebraically

Extra Practice with Answers